Problem 4:

Given, =; = ;

where T = 0.4, α = 0.7387, β = 0.2947, = 13.9684, = 2.3707, = 0.0071

= = (1 + + ) = ( + ) ()

=

= ( ( + ) ()

Putting the given values

= ()( + ) ()

Now separating all the basic terms from = ( ( + ) (), the asymptotic calculation of (, ( + ) and (), is shown below

1. For , || = 20log ()

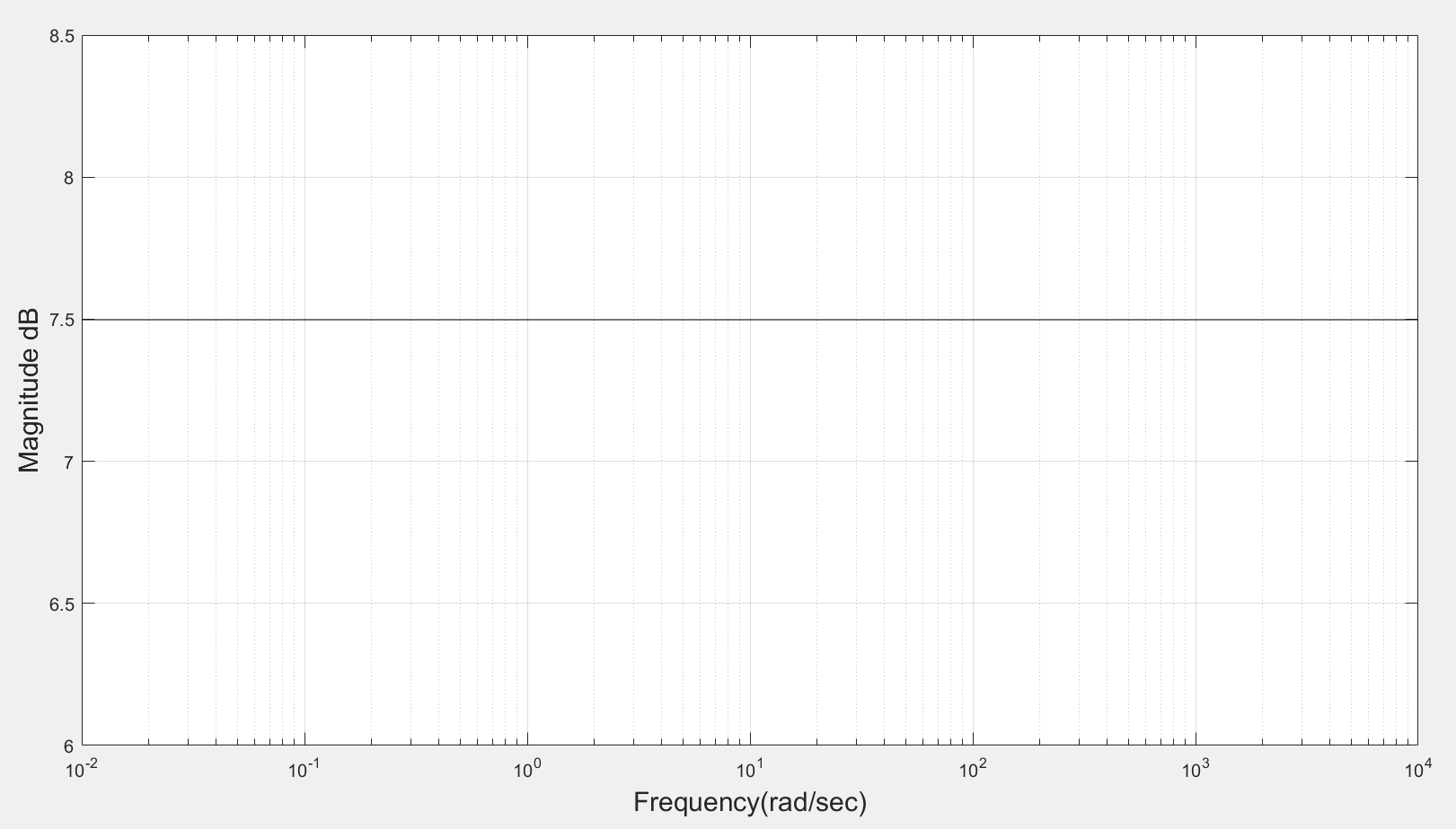


Figure 1: Asymptotic magnitude bode plot for constant gain,

1. For , || = 20log ()

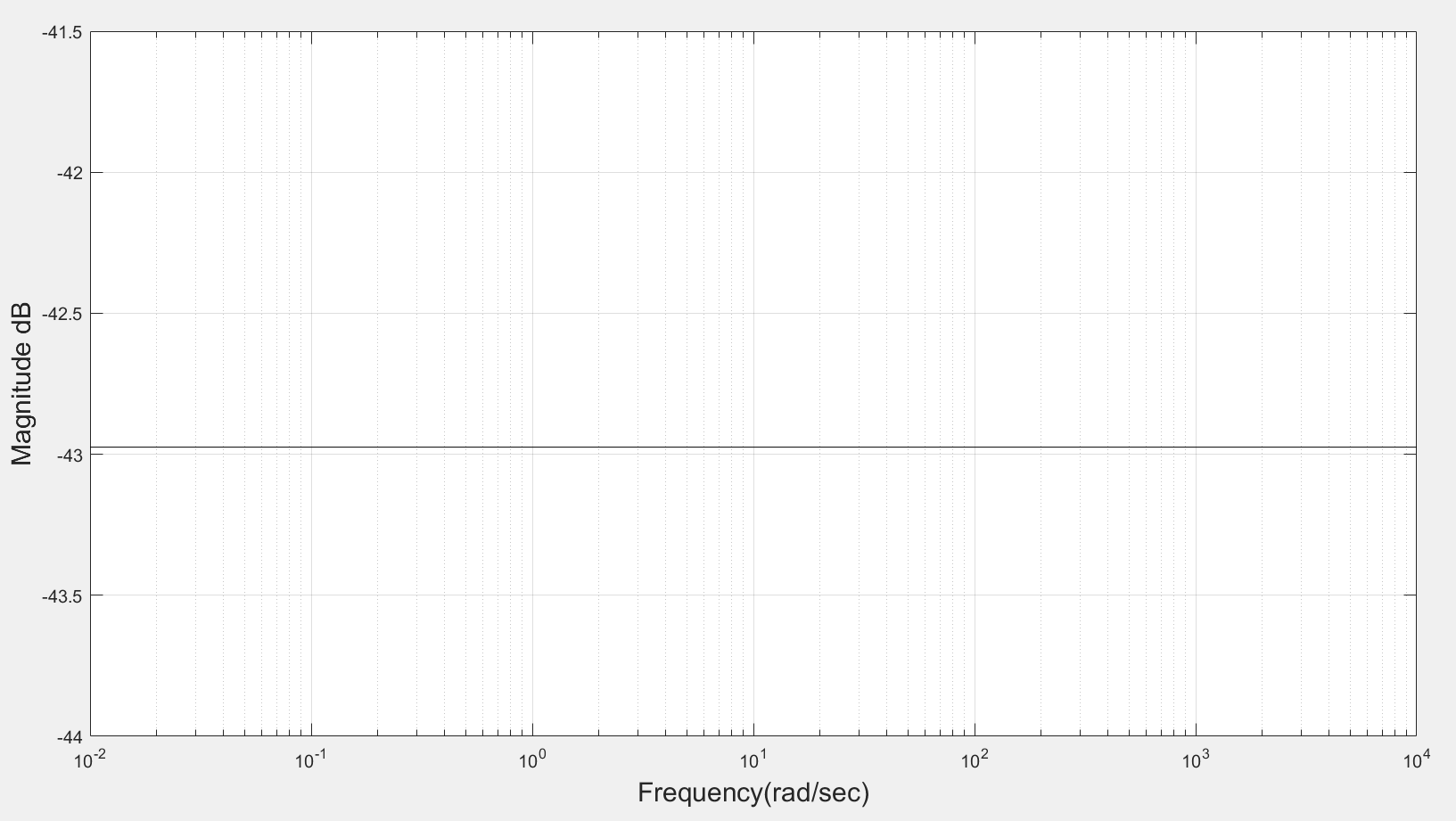


Figure 2: Asymptotic magnitude bode plot for constant gain,

1. For ( + ),

| = ( + ) ……….. (1)

Put s = jω, in equation (1) results into = {( + }……. (2)

**Calculation procedure**

**= .**

= . = .

Applying De Moivre’s theorem in above equation we get

. …… (3)

Again, =. …… (4)

Put equation (3) and (4) in (2) we get

= + j ) + ( + j ) +

= + ) + (j + j )

Magnitude,

|| =

=

=

Magnitude in dB || dB = 20log

In the sum , dominates at lower frequencies whereas dominates at higher frequencies. For approximation we consider = . Now, we obtain corner frequency, =.

Following approximation of magnitude is obtained:

1. For ω ≤ , || dB = 20log||.
2. For ω >, || dB = 20(α+β) log ω.

**Procedure**

* Compute the corner frequency = and locate the point at magnitude

20log| |.

* Draw a slope 0 dB/decade for ω ≤ and a line with slope 20(α+β) dB/decade for

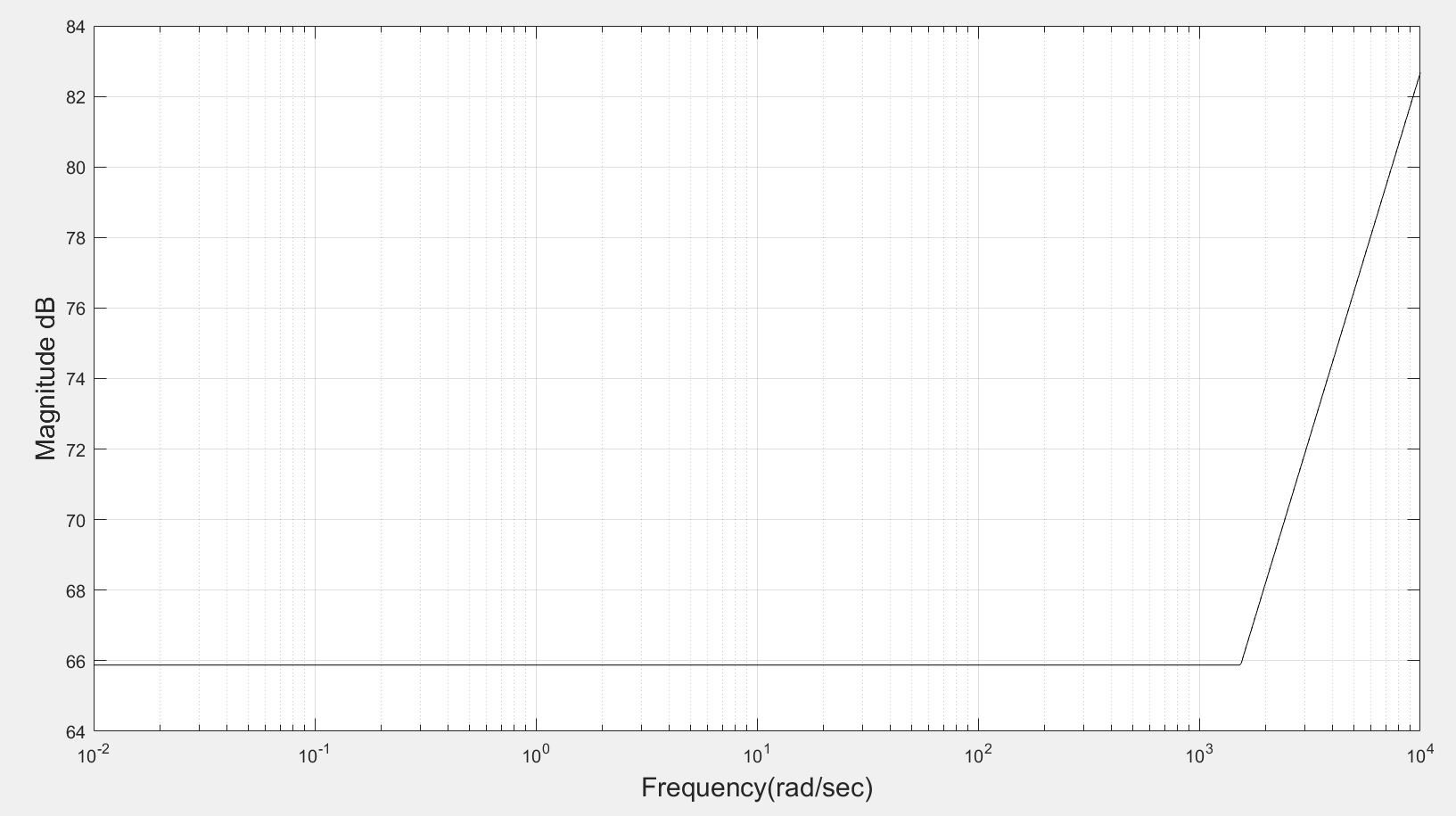


Figure 4: Asymptotic magnitude bode plot for ( + )

1. For,

Fractional zero transfer function is given by =…… (1)

Put s = jω, in equation (1) results into = ……. (2)

Magnitude in dB is given by |)| dB= -20αlogω

**Calculation procedure**

==

Applying De Moivre’s theorem in above equation we get

= . …… (3)

Put equation (3) in (2) we get

T(jω) =

Magnitude, |T(jω) | = =

Magnitude in dB is given by || dB= -20αlogω.

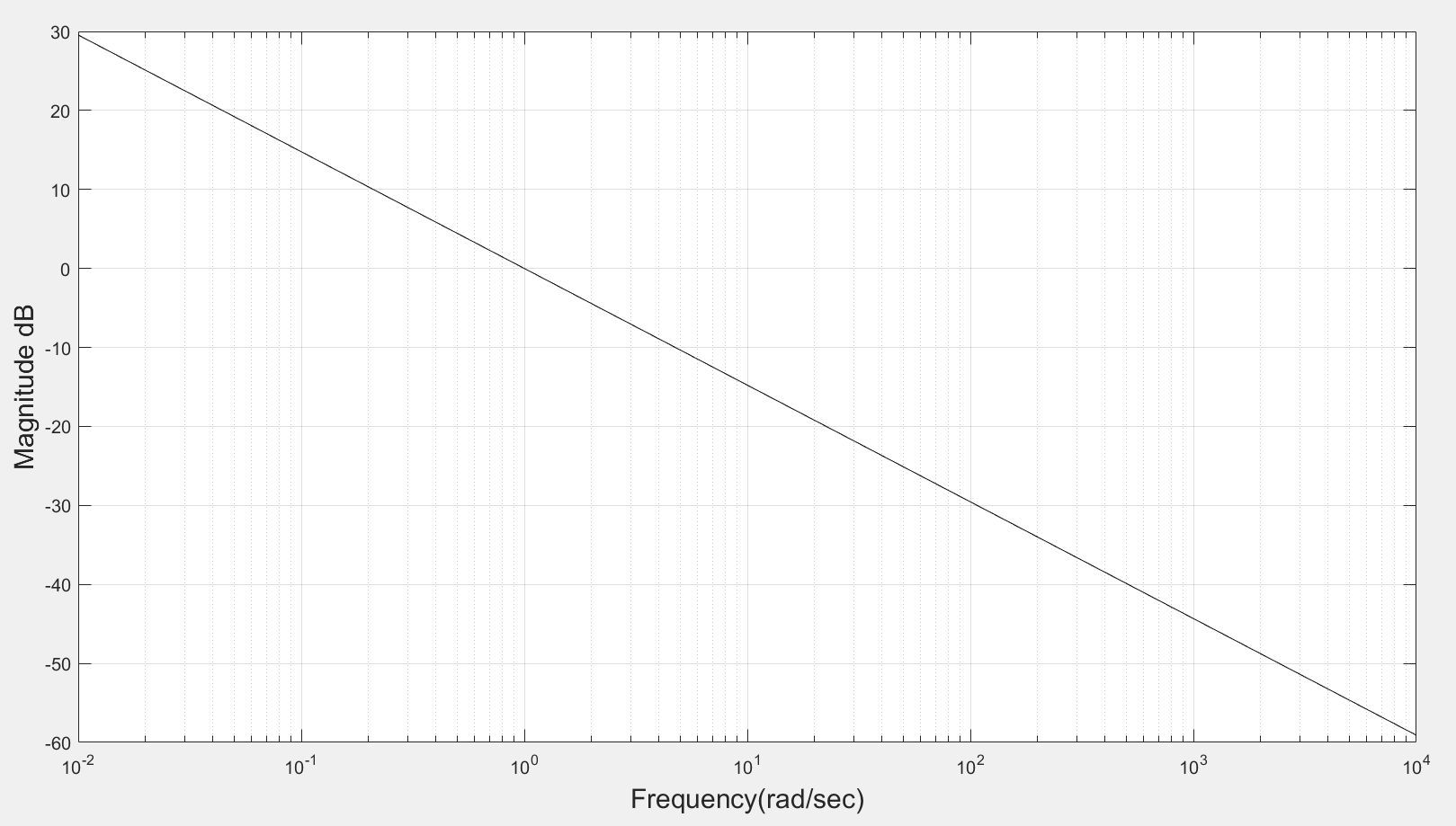


Figure 5: Asymptotic magnitude bode plot for

1. For , putting T = 0.4

T(s) = now put s=jω

T(jω) =--------(a)

Now,

= = ………. (b)

Putting (b) in (a)

= =

Magnitude, || =

=

=

Now, Magnitude in dB, || dB = -20log

In the sum , dominates at lower frequencies whereas dominates at higher frequencies.

For approximation we consider = .We obtain corner frequency, =.

Now, following approximation of magnitude is obtained:

1. For ω ≤ , || dB = -20log = -20log|1|.
2. For ω >, || dB = -20log= -20log ()

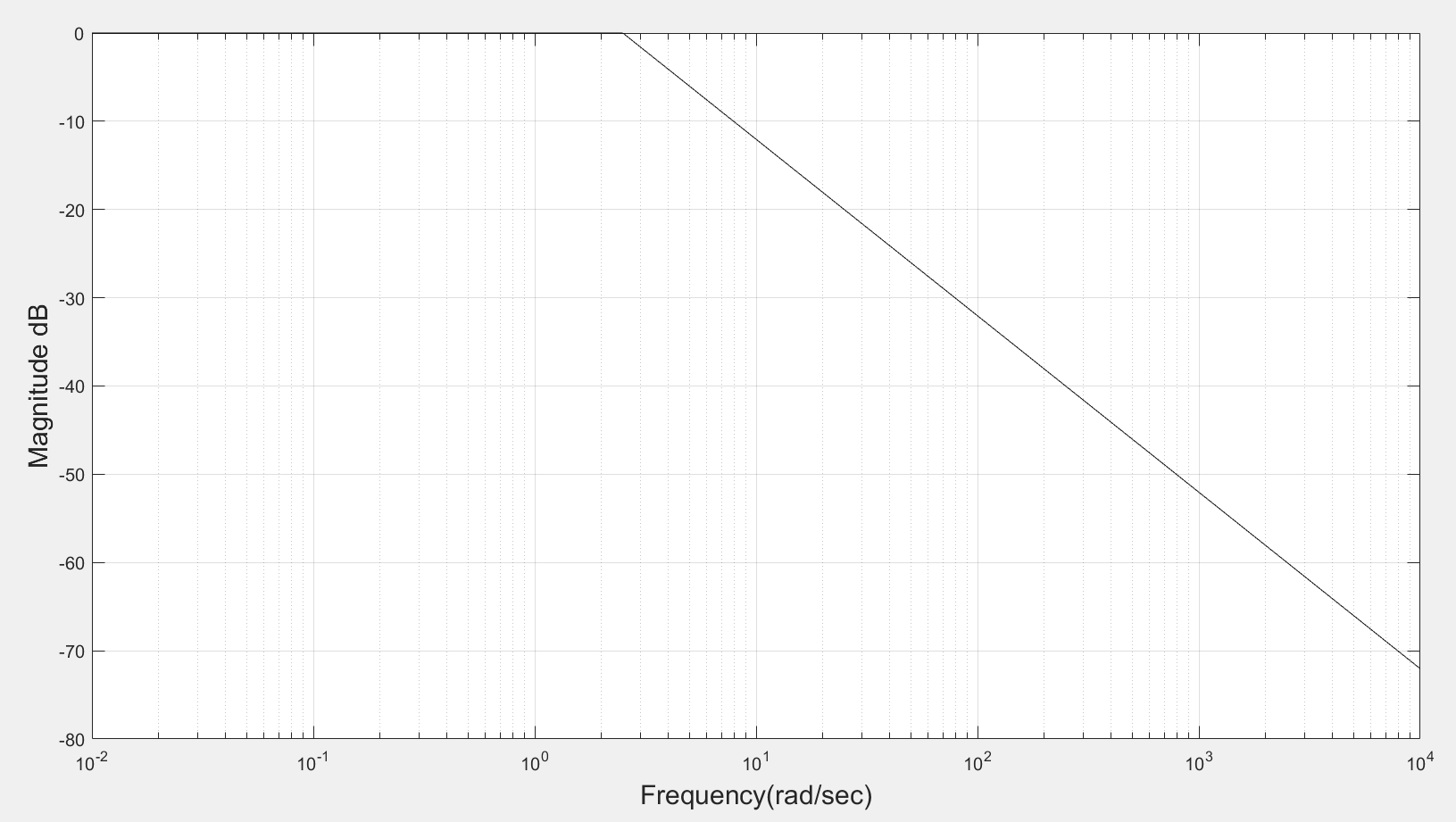


Figure 6: Asymptotic magnitude bode plot for

1. is composed of basic terms, by adding the asymptotic plot of (, ( + ) and (one can find asymptotic plot for

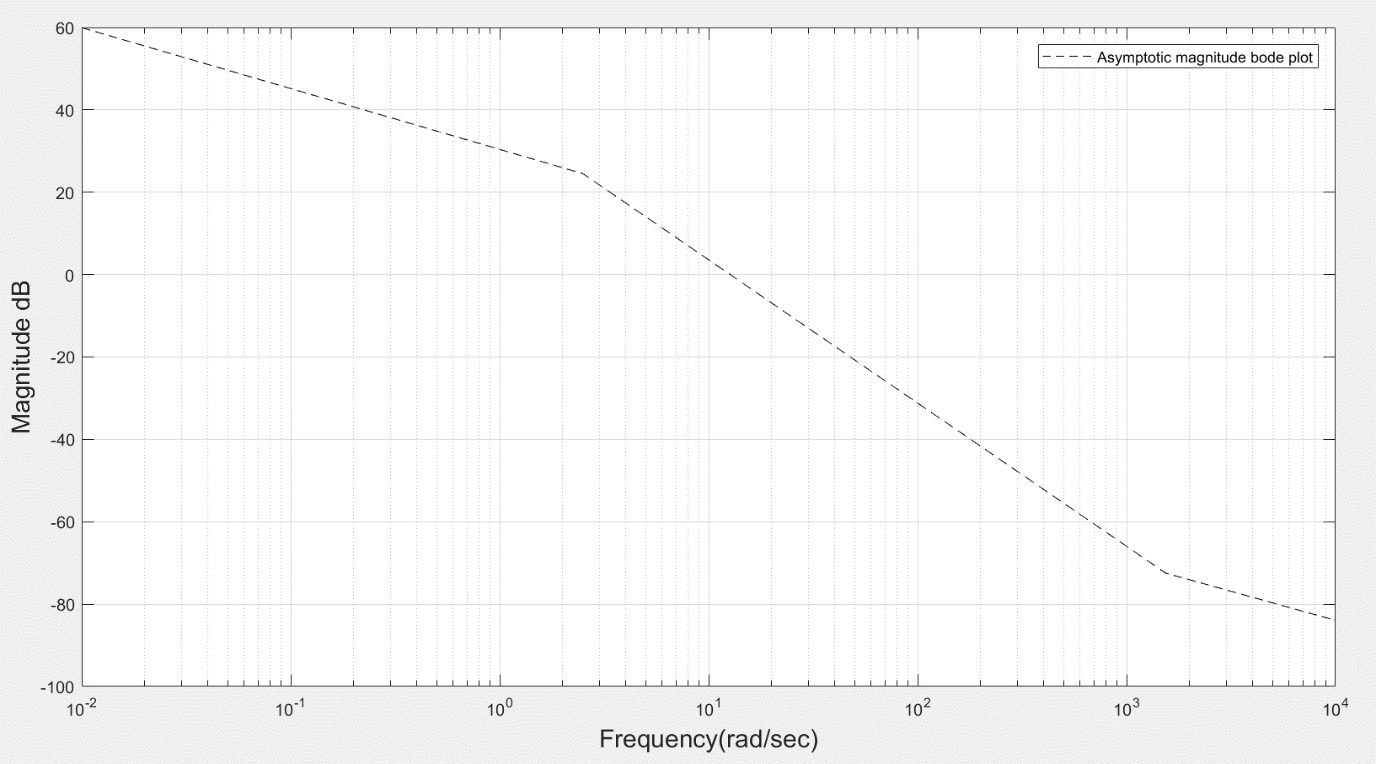


Figure 7: Asymptotic magnitude bode plot for

1. For exact magnitude bode plot of

=

= ( ( + ) ()

Now,

Magnitude, || = 20log{

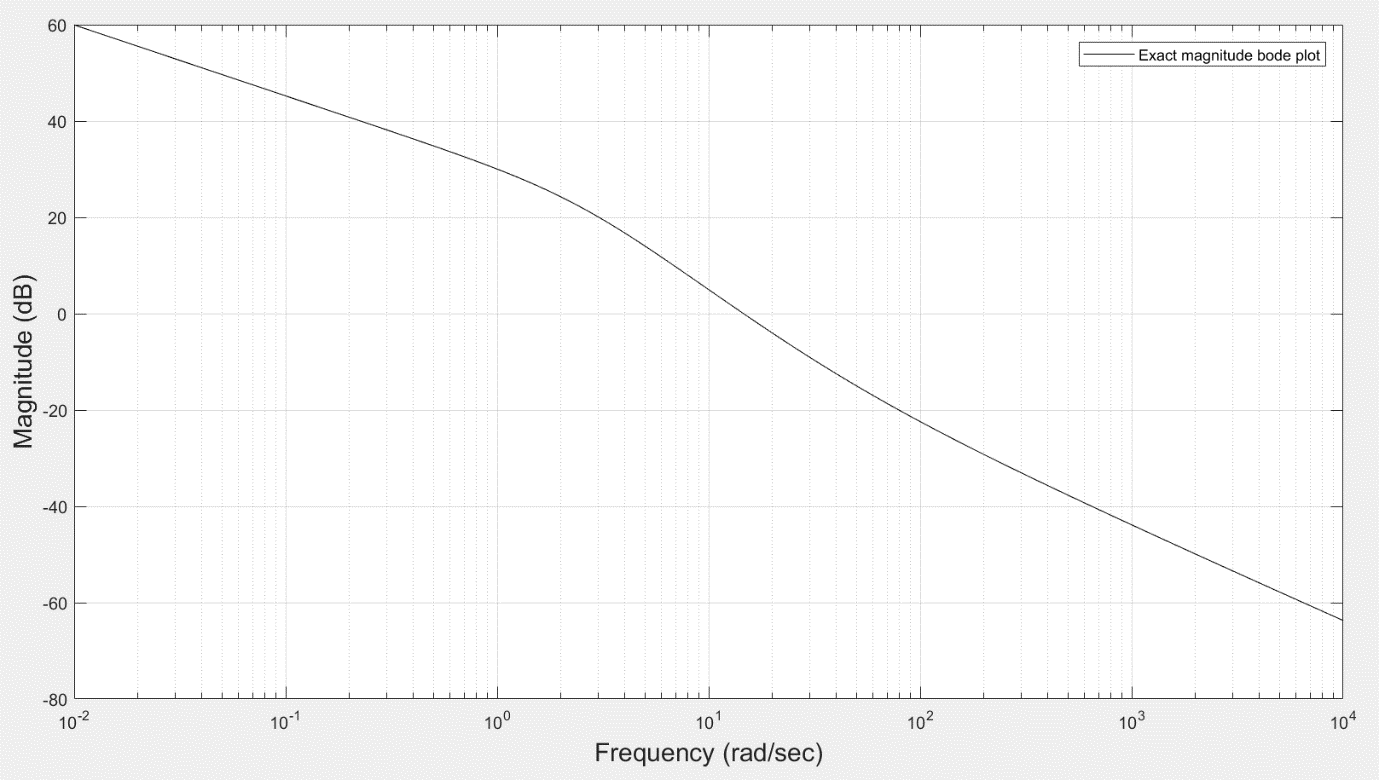


Figure 8: Exact magnitude bode plot for

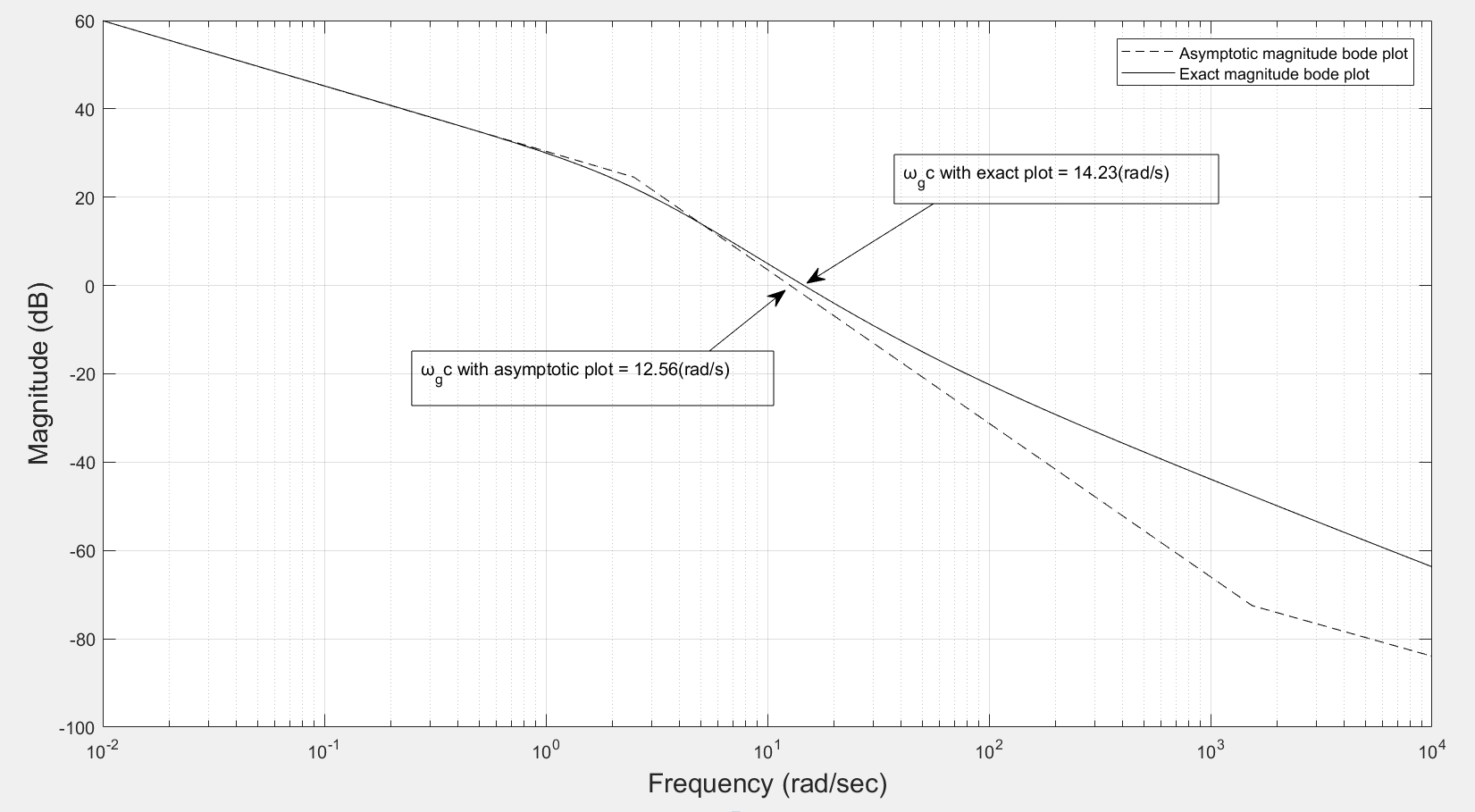


Fig 9: Real and asymptotic magnitude bode plot for

From fig 9, is obtained as

1. for asymptotic = 12.56(rad/sec)
2. for exact plot = 14.23(rad/sec)

**Matlab**

clc;

clf;

clear all;

close all;

Kp = 2.3709;

Ki = 13.9684;

Kd = 0.0071;

alpha = 0.7387;

beta = 0.2947;

T = 0.4;

Wcr = (abs(Ki/Kd))^(1/(alpha + beta));

Wcr1 = (abs(1/0.4));

w = logspace(-2 , 4, 1000);

mask = w < Wcr;

mask1 = w < Wcr1

mag1 = 20\*log10(Kp)\*ones(size(w));

mag2 = 20\*log10(Kd)\*ones(size(w));

mag3 = (mask).\*(20\*log10(Ki/Kd)) + (~mask).\*(20\*(alpha + beta)\*log10(w));

mag4 = -20\*alpha\*log10(w);

mag5 = (mask1).\*(-20\*log10(1)) + (~mask1).\*(-20\*log10(0.4\*w));

combined = mag1 + mag2 + mag3 + mag4 + mag5 ;

figure(1);

plot(1,1);

semilogx(w, mag1,'Color','black')

xlabel('Frequency(rad/sec)','FontSize', 15);

ylabel('Magnitude dB','FontSize', 15);

grid on;

figure(2);

plot(1,2);

semilogx(w, mag2,'Color','black')

xlabel('Frequency(rad/sec)','FontSize', 15);

ylabel('Magnitude dB','FontSize', 15);

grid on;

figure(3);

plot(1,3);

semilogx(w, mag3,'Color','black')

xlabel('Frequency(rad/sec)','FontSize', 15);

ylabel('Magnitude dB','FontSize', 15);

grid on;

figure (4);

plot(1,4);

semilogx(w, mag4,'Color','black')

xlabel('Frequency(rad/sec)','FontSize', 15);

ylabel('Magnitude dB','FontSize', 15);

grid on;

figure (5);

plot(1,5);

semilogx(w, mag5,'Color','black')

xlabel('Frequency(rad/sec)','FontSize', 15);

ylabel('Magnitude dB','FontSize', 15);

grid on;

figure(6);

plot(1,6);

semilogx(w, combined,'Color','black','LineStyle','--')

xlabel('Frequency(rad/sec)','FontSize', 15);

ylabel('Magnitude dB','FontSize', 15);

grid on;

legend('Asymptotic magnitude bode plot','Location','NorthEast')

plot(1,1);

semilogx(w, combined,'Color','black','LineStyle','--')

xlabel('Frequency(rad/sec)','FontSize', 15);

ylabel('Magnitude dB','FontSize', 15);

grid on;

hold on;

clc;

clear

Kp = 2.3709;

Ki = 13.9684;

Kd = 0.0071;

alpha = 0.7387;

beta = 0.2947;

T = 0.4;

w=logspace(-2,4,1000);

Mag=@(w) (20\*log10(abs(sqrt(Kp).^2)) + 20\*log10(abs(sqrt(Kd).^2)) + 20\*log10(abs(sqrt((i.\*w).^(alpha+beta) + ((i.\*w).^alpha)/Kd + (Ki/Kd)).^2)) - 20\*log10(abs(sqrt((T\*(i.\*w)) + 1).^2)) - 20\*log10(abs(sqrt(i\*w).^alpha).^2));

semilogx(w,Mag(w) ,'Color','black');

hold on;

grid on;

xlabel('Frequency (rad/sec)','FontSize', 15);

ylabel('Magnitude (dB)','FontSize', 15);

legend('Asymptotic magnitude bode plot','Exact magnitude bode plot','Location','NorthEast')